v_z = axial velocity, m/s
v_ξ = streamwise velocity, m/s
z = axial coordinate

Greek Letters

ΔP = pressure drop across the HCT, Equation (6), N/m^2
ζ = cos η = transformed coordinate
ζ_0 = HCT coordinate boundary
η = oblate spheroidal coordinate (see Figure 1)
η_I = interception efficiency, Equation (14)
λ = sinhξ = transformed coordinate
λ_0 = HCT coordinate boundary
μ = viscosity of the medium, kg/m s
ξ = oblate spheroidal coordinate (see Figure 1)
ρ = density of the medium, kg/m^3
ψ = stream function, m^3/s

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Interceptional and Gravitational Deposition of Inertialess Particles On a Single Sphere and In a Granular Bed

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Separation of small dust particles on a single sphere or droplet due to such effects as inertia, interception and gravity, has been investigated by Michael and Norey (1969) Flint and Howarth (1971) Prieve and Ruckenstein (1974) and Nielsen and Hill (1976a, b). Deposition of such particles in a granular bed was treated by Rajagopalan and Tien (1976) and Gutfinger and Tardos (1979). Different flow models for the fluid motion around the sphere such as Stokes, Oseen, and potential flows were used.

Based on the work of Nielsen and Hill (1976a), a general mathematical method is applicable to compute interceptional, gravitational (and certain electrical) deposition efficiencies, if the dust particles are very small or move slow enough so that inertial effects can be neglected. The physical meaning of this assumption is that the small dust particles follow the fluid stream lines exactly. In this note, we present a general mathematical approach for the case of deposition on a single sphere and/or spherical particle (granule) situated in a packed or densely fluidized bed. Also, the meaning of "inertia-less particles" is tested, using a numerical solution for the dust particle trajectory near the surface of the sphere of granules.

As a first step in computing the deposition efficiency, one has to know the stream function characteristic for the fluid flow around a sphere ψ_f, and the stream function of the gravity force field, ψ_G. These functions given by Nielsen and Hill (1976a) are

$$\psi_f = \frac{1}{2} R^2 \sin^2\theta \, h(R, \epsilon) \tag{1}$$

$$\psi_G = \frac{1}{2} R^2 \sin^2\theta \, GaSt \tag{2}$$

Here, $R = r/a$, is the dimensionless radius, $Ga = ag/U_0^2$ is the Galileo number, $St = 2C\rho_p U_0 r^2_p/9\mu a$ is the Stokes number (or

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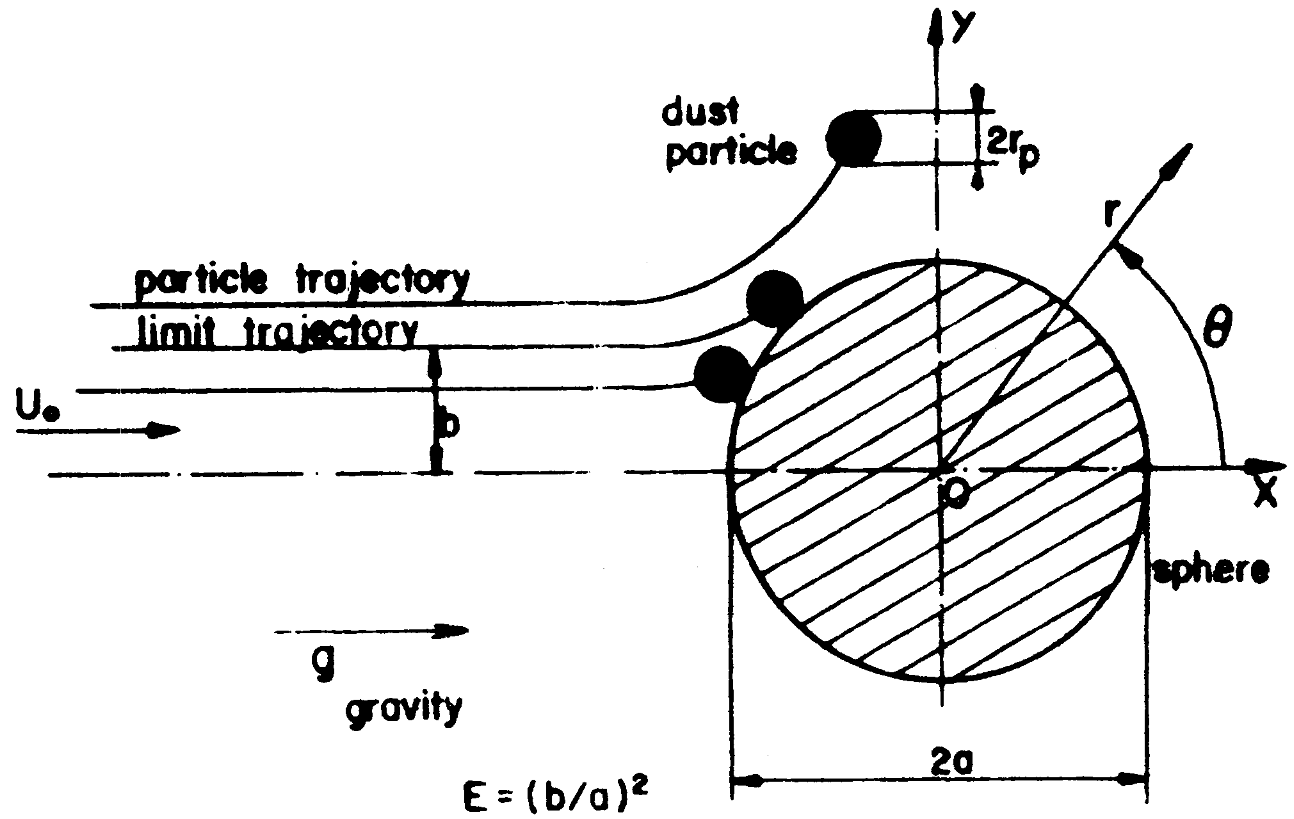

Figure 1. Deposition of Small Particles on a Sphere.

inertial parameter), and $h(R, \epsilon)$ is a coordinate and bed porosity, ϵ, dependent function given in Table 1. The assumption in Equation (2) is that the gravity field is parallel to the flow and is in the same direction, as shown in Figure 1. The limiting conditions for $h(R, \epsilon)$ are

$$\begin{aligned} h(R, \epsilon) &= 1 \quad R \rightarrow \infty \\ h(R, \epsilon) &= 0 \quad R = 1 \end{aligned} \tag{3}$$

Figure 1 shows the deposition of particles on a single sphere. The limiting trajectory is defined as the last trajectory from the OX axis along which a dust particle travels and just touches the sphere and is thus intercepted. The single particle deposition efficiency is

$$E = (b/a)^2 = B^2 \tag{4}$$

where, B is the dimensionless distance of the limiting trajectory from the OX axis far upstream the sphere. Since the dust particles are considered to be inertia-less ($St \rightarrow 0$), their motion deviates from the fluid streamlines only due to the gravity force. Thus, we can show (see Appendix) that the stream function describing the trajectory of the dust particles is given by

$$\psi = \psi_f + \psi_G = \frac{1}{2} R^2[GaSt + h(R, \epsilon)] \sin^2\theta \tag{5}$$

The value of the stream function containing the point $X_0(R \rightarrow \infty, \sin\theta = Y/R, \cos\theta = -1)$ situated far upstream from the sphere is

$$\psi|_{X_0} = \frac{1}{2} Y^2[1 + GaSt] \tag{6}$$

Equating (5) and (6), one obtains the equation of the stream lines passing through X_0 as

$$Y^2 = R^2 \sin^2\theta \frac{GaSt + h(R, \epsilon)}{1 + GaSt} \tag{7}$$

The equation of the limiting trajectory is obtained when $Y = B$ and

$$R = 1 + r_p/a = 1 + R_p \tag{8}$$

TABLE 1. VALUES OF THE FUNCTION $h(R, \epsilon)$.

$h(R, \epsilon)$	Flow Field Model	Author	Remarks
$1 - \frac{1}{R^3}$	Potential flow	Lamb (1932)	single sphere, $\epsilon = 1$
$1 - \frac{3}{2}\frac{1}{R} + \frac{1}{2}\frac{1}{R^3}$	Stokes or creeping flow	Lamb (1932)	single sphere, $\epsilon = 1$
$\frac{1}{\epsilon}\left(1 - \frac{1}{R^3}\right)$	Potential flow	Lamb (1932)	granular bed; near the surface of the granule
$\approx \frac{3}{2} g(\epsilon)^3\left(\frac{R-1}{R}\right)^2$*	Stokes flow	Tardos et al. (1976 and 1978)	granular bed; near the surface of the granule; approximate solution

*$g(\epsilon)$ is a porosity dependent function deduced first by Pfeffer (1964) using the Happel cell model. Value of this function can be taken with good approximation as $g(\epsilon) = 1.31/\epsilon$, as shown by comparison with experimental data by Tardos et al. (1976, 1978).

TABLE 2. INTERCEPTIONAL DEPOSITION EFFICIENCY.

E_R	E_R Approximate expression for small values of R_p	Flow Field	Remarks
$(1 + R_p)^2 - 1.5(1 + R_p) + \frac{0.5}{1 + R_p}$	$1.5R_p^2 - 0.5R_p^3 + 0(R_p^4)$	Stokes	single sphere
$(1 + R_p)^2 - \frac{1}{1 + R_p}$	$3R_p + R_p^3 - 0(R_p^4)$	Potential	single sphere
	$1.5g(\epsilon)^3R_p^2 - 0(R_p^3)$	Stokes	granular bed
	$\frac{3}{\epsilon} R_p + 0(R_p)^3$	Potential	granular bed

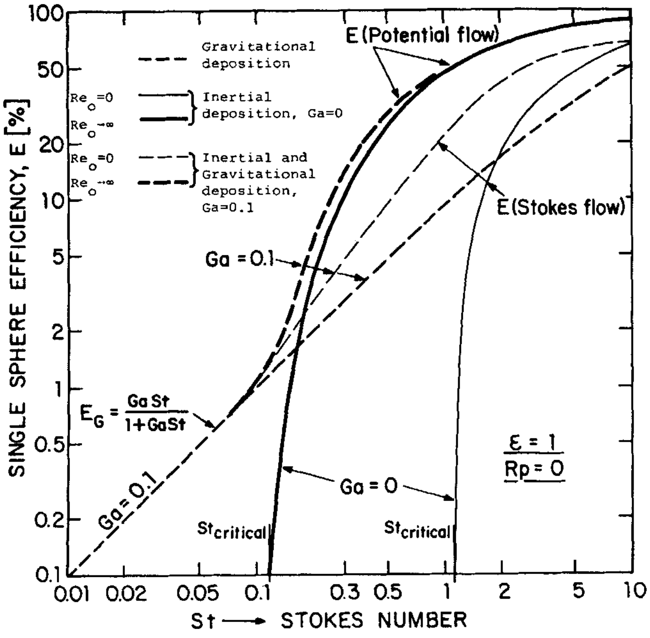

Figure 2. Inertial and Gravitational Deposition of Small Particles on a Single Sphere (ϵ = 1) for R_p = 0.

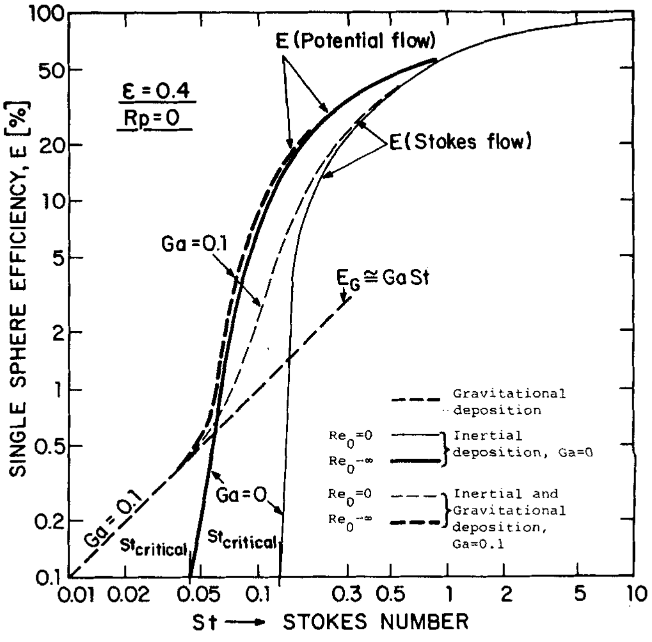

Figure 3. Inertial and Gravitational Deposition of Small Particles on a Granule in a Granular Bed of Porosity ϵ = 0.4 for R_p = 0.

where, R_p is the interception parameter. The physical meaning of the condition (8) is that each particle approaching the collecting sphere at a distance equal or less than its radius is intercepted. Using Equation (4) together with (7) and (8), one obtains for the single particle collection efficiency, E, the expression

$$E(\theta) = B^2 = (1 + R_p)^2 \sin^2\theta \, \frac{GaSt + h(R, \epsilon)|_{R=1+R_p}}{1 + GaSt} \tag{9}$$

Since we are interested in the maximum value of the efficiency, E, Equation (9) becomes

$$E_{RG} = (1 + R_p)^2 \, \frac{GaSt + h(R, \epsilon)|_{R=1+R_p}}{1 + GaSt} \tag{10}$$

where, E_{RG}, is the efficiency due to both direct interception and gravitational effects. The two limiting cases, that of pure interception, E_R (for Ga = 0) and that of pure gravitational separation, E_G (for R_p = 0) are obtained as

$$E_R = (1 + R_p)^2 \, h(R, \epsilon)|_{R=1+R_p} \tag{11}$$

and

$$E_G = \frac{GaSt}{1 + GaSt} \tag{12}$$

The combined efficiency E_{RG}, is

$$E_{RG} = (1 + R_p)^2 E_G + \frac{E_R}{1 + GaSt} \tag{13}$$

Two important conclusions may be drawn at this point. First, the gravitational and interceptional deposition efficiencies are not additive unless R_p and Ga are negligibly small. Second, the deposition efficiency due to gravitational effects E_G (Eq. 12), is independent of the flow field and therefore independent of the granular bed porosity, ϵ. This is true for any flow field for which $h(1, \epsilon) = 0$. Thus the expression of Rajagopalan and Tien (1976) for gravitational deposition efficiencies in a bed of particles is in error and the result obtained by Prieve and Ruckenstein (1974)

$$E_G = GaSt \tag{14}$$

is only an approximation. Table 2 presents different expressions for the interceptional deposition efficiency E_R as obtained from Eq (11) and Table 1.

To verify our second conclusion that E_G is independent of both flow field and bed porosity, the dust particle trajectory was numerically computed for a constant value of the Galileo number, Ga, for decreasing values of the Stokes number. The procedure for such computations is described in Tardos et. al. (1974) and Nielsen and Hill (1976b). Results are presented in Figures 2 and 3 for two cases, that of deposition on a single sphere (ϵ = 1) and in a granular bed (ϵ = 0.4) for zero interception (R_p = 0). In both cases, the combined efficiency due to inertia and gravity reaches the analytical value E_G at sufficiently low Stokes numbers (approximately St = 0.05). The same result was reached by Michael and Norey (1969) for the case of deposition on a single sphere in potential flow and by Flint and Howarth (1971) for the case of single gas bubbles in potential and Stokes flow.

On the basis of Figures 2 and 3, we may state that for values of the Stokes number smaller than about St = 0.05, the dust particles can be considered inertia-less. For this situation, and in the absence of collection by diffusion (Brownian motion), electrostatics, etc., the analytical value of the efficiency E_{RG} as given by Eq. (13) may be used. But for higher values of the Stokes number, a combined inertial, interception and gravitational efficiency, must be computed numerically using the concept of the limiting trajectory as in Figures 2 and 3.

For the special case of gravitational effects acting against inertia (negative Galileo numbers), the combined deposition efficiency E_{IG} is decreased, resulting in a lower critical Stokes number, St_{critical}, under which no collection occurs. Computations of this kind were first performed by Michael and Norey (1969), for deposition on a single sphere in potential flow. The general combined inertial, interception and diffusional deposition in granular beds under different flow conditions (including electrostatic effects) was presented by the authors in two recent papers (Gutfinger and Tardos 1979 and Tardos and Pfeffer 1979).

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NOTATION

a	= sphere (collector, grain) radius
b	= vertical distance of the limit trajectory
$B = b/a$	= dimensionless distance, in Eq (4)
C	= Cunningham correction factor
$h(R, \epsilon)$	= function given in Table 1

g = acceleration of gravity
$Ga = ag/U_0^2$ = Galileo number
$g(\epsilon)$ = porosity dependent function, in Table 1
E = single sphere efficiency
E_R = single sphere efficiency due to interception
E_G = single sphere efficiency due to gravity
E_{RG} = single sphere efficiency due to interception and gravity
E_{IG} = single sphere efficiency due to inertia and gravity
r = radial coordinate
r_p = dust particle radius
R = dimensionless radius
$R_p = r_p/a$ = interception parameter
$Re_0 = 2aU_0/\nu$ = Reynolds number
$St = 2C\rho_p U_0 r_p^2/9\mu a$ = Stokes number
$St_{critical}$ = critical Stokes number
X, Y = dimensionless coordinates
X_0 = point situated for upstream of the granule
U_0 = flow superficial velocity

Greek Letters

ψ_f = dimensionless stream function for fluid flow
ψ_G = dimensionless stream function for gravity field
ϵ = bed porosity
ρ_p = dust particle density
μ = gas viscosity
θ = coordinate
ν = gas kinematic viscosity

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APPENDIX

The dimensionless equations of motion for a dust particle in the presence of a gravitational force acting in the positive X direction (see Figure 1) are given (Michael and Norey 1969) as

$$St \frac{dV_y}{dt} = U_y - V_y \tag{A1}$$

$$St \frac{dV_x}{dt} = U_x - V_x + GaSt \tag{A2}$$

where V represents the velocity of the dust particle and U the velocity of the fluid. For "inertia-less" particles, let the Stokes number, St, approach zero ($St \to 0$), but keep the characteristic gravity term, $GaSt$, constant ($Ga \to \infty$). Equation (A1) becomes simply

$$V_y = U_y \tag{A3}$$

To examine the behavior of Equation (A2) we set

$$Q = V_x - U_x \tag{A4}$$

and write (A2) as

$$St \frac{dQ}{dt} = -Q - St\left(\frac{dU_x}{dt} - Ga\right) \tag{A5}$$

As $St \to 0$, the characteristic time of this first order linear equation in Q becomes very short, therefore the steady state is quickly reached so that

$$Q + St\left(\frac{dU_x}{dt} - Ga\right) = 0 \tag{A6}$$

It can be easily verified that dU_x/dt is of order unity, therefore as $Ga \to \infty$, Equation (A6) becomes

$$V_x = U_x + GaSt \tag{A7}$$

Equations (A3) and (A7) can be rewritten in spherical coordinates as

$$\begin{aligned} V_\theta &= U_\theta - GaSt \sin\theta \\ V_r &= U_r + GaSt \cos\theta \end{aligned} \tag{A8}$$

If we choose Stokes flow around a single sphere, for example, then

$$\begin{aligned} U_\theta &= -\left[1 - \frac{3}{4}\frac{1}{R} - \frac{1}{4}\frac{1}{R^3}\right]\sin\theta \\ U_r &= \left[1 - \frac{3}{2}\frac{1}{R} + \frac{1}{2}\frac{1}{R^3}\right]\cos\theta \end{aligned} \tag{A9}$$

so that Equations (A8) become

$$\begin{aligned} V_\theta &= -\left[1 + GaSt - \frac{3}{4R} - \frac{1}{4R^3}\right]\sin\theta \\ V_r &= \left[1 + GaSt - \frac{3}{2R} + \frac{1}{2R^3}\right]\cos\theta \end{aligned} \tag{A10}$$

Using the usual relationship between the stream function and the velocity components

$$V_\theta = -\frac{1}{R\sin\theta}\frac{\partial\psi}{\partial R} \tag{A11}$$

$$V_r = \frac{1}{R^2\sin\theta}\frac{\partial\psi}{\partial\theta} \tag{A12}$$

Equation (A10) can be rewritten in terms of the stream function ψ which describes the dust particle trajectories as

$$\psi = \frac{1}{2}R^2\left[1 + GaSt - \frac{3}{2R} + \frac{1}{2R^3}\right]\sin^2\theta \tag{A13}$$

where for this particular case (Table 1)

$$h(R, \epsilon) = 1 - \frac{3}{2R} + \frac{1}{2R^3} \tag{A14}$$

so that:

$$\psi = \frac{1}{2}R^2[GaSt + h(R, \epsilon)]\sin^2\theta \tag{A15}$$

which is Equation (5).

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